Game Theory Questions

Math 20
Due October 18, 2004

1. Suppose that a game has a payoff matrix

$$A = \begin{bmatrix} -4 & 6 & -4 & 1 \\ 5 & -7 & 3 & 8 \\ -8 & 0 & 6 & -2 \end{bmatrix}$$

   (a) If players *R* and *C* use strategies

$$\mathbf{p} = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}; \mathbf{q} = \begin{bmatrix} \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \end{bmatrix}$$

   what is the expected payoff of the game?

   (b) If player *C* keeps his strategy fixed as in part (a), what strategy should player *R* choose to maximize his expected payoff?

   (c) If player *R* keeps her strategy fixed as in part (a), what strategy should player *C* choose to maximize the expected payoff to player *R*?

2. Construct a simple example to show that optimal strategies are not necessarily unique. For example, find a payoff matrix with several equal saddle points

3. For the strictly determined games with the following payoff matrices, find optimal strategies for the two players, and find the values of the games.

   (a) $\begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$

   (b) $\begin{bmatrix} -3 & -2 \\ 2 & 4 \\ -4 & 1 \end{bmatrix}$

   (c) $\begin{bmatrix} 2 & -2 & 0 \\ -6 & 0 & -5 \\ 5 & 2 & 3 \end{bmatrix}$

(d) $\begin{bmatrix} -3 & 2 & -1 \\ -2 & -1 & 5 \\ -4 & 1 & 0 \\ -3 & 4 & 6 \end{bmatrix}$

4. For the 2x2 games with the following payoff matrices, find optimal strategies for the two players, and find the values of the games

(a) $\begin{bmatrix} 6 & 3 \\ -1 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 40 & 20 \\ -10 & 30 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & 7 \\ -5 & 4 \end{bmatrix}$

(d) $\begin{bmatrix} 3 & 5 \\ 5 & 2 \end{bmatrix}$

(e) $\begin{bmatrix} 7 & -3 \\ -5 & -2 \end{bmatrix}$

5. Player *R* has two playing cards: a black ace and a red four. Player *C* also has two cards: a black two and a red three. Each player secretly selects one of his or her cards. If both selected cards are the same color, player *C* pays player *R* the sum of the face values in dollars. If the cards are different colors, player *R* pays player *C* the sum of the face values in dollars. What are optimal strategies for both players, and what is the value of the game?